

Core function

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Abstract. Let Π_X be the class of Lebesgue dominated probability measures on (Borel sets of) the real line R supported by $\emptyset \neq X = (a, b) \subseteq R$, and satisfying standard regularity conditions. Score function S of distribution $G \in \Pi_R$ with density $g(u)$ is defined as

$$S(u) = -\frac{d \ln g(u)}{du}. \quad (1)$$

Consider the location-scale family $G_{(\mu, \sigma)} \in \Pi_R$ with parent G and densities in the form $g_{(\mu, \sigma)}(y) = \sigma^{-1}g(u)$ where

$$u = \frac{y - \mu}{\sigma} \quad (2)$$

is the pivotal variable (Fraser, 1968). The *core function* of $G_{(\mu, \sigma)}$ was defined (Fabián, 2001) by formula (1) with u given by (2). Even in the case of the normal distribution, the core function is different from the score. Indeed, $S(u) = u$ whereas the score function $-\partial \log g_{(\mu, \sigma)}(y)/\partial y = u/\sigma$.

Let $X \neq R$ and $F \in \Pi_X$. Let $F = G\eta$ where $\eta^{-1} : R \rightarrow X$ is a generalized Johnson mapping (Johnson 1949). A parametric family $F_{(\tau, \sigma)}(x) = G_{(\mu, \sigma)}(\eta(y))$ with parent $F = G\eta$ has densities $f_{(\tau, \sigma)}(x) = g(u)\eta'(x)$, where u is the pivotal variable,

$$u = \frac{\eta(x) - \eta(\tau)}{\sigma}, \quad (3)$$

where $x = \eta^{-1}(y)$ and $\tau = \eta^{-1}(\mu)$ is the 'Johnson location'. Even in this case, the core function of distribution $F_{(\tau, \sigma)}$ is given by the negative derivative of the logarithm of density (1) according, however, to pivotal variable (3).

Johnson parametrized families contain many important statistical models and Johnson location is an alternative measure of central tendency of a distribution (it always exists, in contrast to the mean). In the paper we show how the core functions can be used for solution of parametric statistical inference problems.

References

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