

Estimating IRT Abilities using Weighted Observed Score

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Abstract

When the outcome of a multinomial distribution is scored, we call it an scored multinomial distribution. Strictly speaking, a test score can be considered to have a scored multinomial distribution.

In the first part of this presentation, the distribution of the integer-weighted sum of two scored multinomial distributions is derived. The result is presented as a theorem, and through its recursive applications, the sum of several scored multinomial distributions can be calculated. The compound binomial distribution can be considered as a special case of this distribution where each of the multinomial distribution has only two categories and equal weight. The algorithm is easy to apply and very fast.

In the second part, an IRT application is presented. Under IRT, it is known that, given θ , the distribution of the number right score is a compound binomial distribution and that the posterior distribution of θ can be calculated on the basis of this conditional distribution. Using the results of the first part, it is possible to derive the distribution of an integer-weighted observed score as a sum of scored multinomial distributions. Given the conditional distribution of the weighted observed score, the inference on θ can be made by calculating the posterior distribution of θ given the observed score.

Several types of scoring weights are compared with regard to the efficiency of estimating θ .