Asymmetric multidimensional scaling based on joint space model

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Abstract
A model and an associated algorithm for two-mode three-way asymmetric proximities (object×object×source), which were extended from Okada and Imaizumi (2003), are introduced. A joint space model, where both objects and sources are represented in the same multidimensional space, is used. Object j is represented as a point and a circle (sphere, hypersphere) of radius j, and source i is represented as a point called the dominance point (c.f., Krumhansl,1978). The model consists of the common joint configuration, the symmetry weight configuration, the asymmetry weight. Let s_{jki} (similarity or dissimilarity) be the proximity from objects j to k for source i. s_{jki} is assumed to be monotonically decreasingly or increasingly related to m_{jki}

$$m_{jki} = d_{jki} - (1 - \beta_i exp(-d_{ij}^2 c))r_j + (1 - \beta_i exp(-d_{ik}^2 c))r_k,$$

where d_{ij} is the distance between the points representing object j and source i in the common joint configuration, $\beta_i (\geq 0)$ is the asymmtry weight for source i, c is the constant to regulate the effect caused by d_{ij}, and d_{jki} is

$$d_{jki} = \sqrt{\sum_{s=1}^{p} w_{is}(x_{js} - x_{ks})^2}.$$

w_{is} is the symmetry weight for source i along dimension s, and x_{js} is the coordinate of object j along dimension s in the common joint configuration. The model has three characteristics when compared with Okada and Imaizumi (2003); (a) the orientation of dimensions of the common joint configuration is uniquely determined up to reflections and permutations, (b) more subtle aspects of differences among sources on symmetric proximity relationships among objects are presented, and (c) differences among sources can be relateded to the uniquely determined dimensions.

References

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