A New Method for Testing Univariate and Bivariate Normality Based on Properties of Hermite Orthogonal Polynomials

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Keywords: Normal distribution, hypothesis testing, orthogonal polynomials, Monte-Carlo simulation, Shapiro-Wilk test.

Abstract

A new approach to testing univariate normality is proposed. The approach is based on the properties of Hermite orthogonal polynomials. If x is a normally distributed random scalar, then, due to the orthonormality relations, the correlation coefficient between Hermite polynomials is unity for the polynomials of the same order and zero for polynomials of different orders. For any non-normal distribution the orthonormality relations do not hold. Based on this property, a criterion for testing normality is developed (h1-test). The power of this criterion has been evaluated using Monte-Carlo simulation. Four families of alternative distributions are considered: asymmetric, symmetric leptokurtic, symmetric platycurtic and symmetric mesokurtic. The power of the h1-test is compared with the most powerful of known omnibus tests, the Shapiro-Wilk-Francia (SWF) test. It is found that in all the cases considered the h1-test has a comparable or superior power compared with the SWF test. Advantages of the h1-test are especially evident in the case of symmetric mesokurtic alternatives. Unlike the SWF test, the h1-test does not have any limitations on sample size.

A potential of using orthogonal polynomials for testing bivariate normality is explored. If $\{x, y\}$ is a pair of gaussian random variables with correlation coefficient ρ, then the correlation coefficient between the corresponding Hermite polynomials $H_m(x)$ and $H_n(y)$ is a simple analytical function of ρ. Using this function, a criterion for testing bivariate normality is constructed (h2-test). This criterion is sensitive only to the fact of bivariate normality, regardless the particular value of ρ. Power of the h2-criterion has been estimated using Monte-Carlo simulation and the family of 2D gamma distributions as the alternatives. It is shown that the h2-test is highly sensitive even to a slight departure from bivariate normality. Similar to the one-dimensional case, the h2-test is very simple for computations and does not have any limitations on the sample size. Power computations for the h2-test demonstrate promising results.