On Generalized Canonical Correlation Analysis among M Sets of Variables

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Abstract

Jewell and Bloomfield (1983) showed that two sets of variables with positive definite covariance matrix V have precisely the same canonical correlations as the two sets of variables X and Y with the covariance matrix equal to the inverse of V. Baksalary, Puntanen, and Yanai (1992) extended the results to the case where V is singular. Further, they showed that canonical correlations between X and Y, which we denote by $cc_i(X, Y)$, is equal to canonical correlations between $Q(X)Y$ and $Q(Y)X$, where the former is the matrix of residuals obtained by regressing Y onto X, and the latter by regressing X onto Y. $Q(X) = I_n - P(X)$, where $P(X)$ is the orthogonal projector onto the subspace spanned by column vectors of X. Yanai and Puntanen (1993) extended these results to the case where the given set of variables are partitioned into three sets (X, Y, and Z) and showed that partial canonical correlation coefficients between X and Y eliminating Z can be obtained from the inverse of the covariance matrix, and that they are equal to the canonical correlation between $Q(Y, Z)X$ and $Q(X, Z)Y$. In this paper, we define generalized canonical correlation following Yanai (1988), where given sets of variables are partitioned into more than three sets and consider various representations of the generalized canonical correlation coefficients among the extended anti-image variables whose covariance matrix may be singular. (See Yanai and Mukherjee, 1987.)

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