Use of nonnormality in SEM

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Abstract

Structural equation modeling (SEM) is a statistical approach that integrates factor analysis and path analysis (e.g. Bollen, 1989). SEM has been successfully applied to experimental and observational studies in social sciences.

SEM is built on normal assumption and the assumption limits its applicability. It should be noted that observed variables in social sciences are almost never normally distributed. In many cases, the nonnormal distribution is approximated by a normal distribution in the framework of SEM. This implies that higher-order moments are not useful and that the only first- and second-order moments are informative. Use of nonnormality will make it possible to handle a wider variety of models than in the conventional SEM. Mooijaart (1985) studied the generalized least squares estimation using the second- and third-order moments for the model with nonnormal independent factors. He showed that factor rotation is unnecessary in his framework.

Assuming nonnormality of latent variables, we use higher-order moments to extend SEM. We take the least squares approach to estimate model parameters, and we employ Shimizu and Kano's (2003) approach (Browne's statistic) to evaluate the fit of a model statistically. The extension enables us to make the following four statistical inferences that SEM can not handle:

1. a measurement model with single common factor and two indicators can be estimated;
2. the direction of a path in SEM-equivalent models can be determined;
3. regression on common factors and specific factors can be performed;
4. nonlinear correlation between factors can be examined.

Real examples are presented to illustrate the new models developed here.

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