

# Globally Optimal Nonexhaustive Partitioning by Dynamic Programming

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## Abstract

Nonexhaustive partitioning problems may arise in at least two psychometric areas. First, nonexhaustive clustering proposes to cluster only a subset of the objects under study. Although such approaches appear to be more popular in pattern recognition (Theodoridis and Koutroumbas, 1999) and bioinformatics (de Bruijn, 1998), where they are combined with fast sequential algorithms, they may also be used in psychometrics. Objectives for nonexhaustive clustering might be the need of a few small prototypical clusters, or the avoidance of outlier influence on the clusters solution and it's quality assessment. Secondly, nonexhaustive partitioning problems also arise in optimal test assembly (Van der Linden, 1998), where the goal is to form multiple small tests from an item bank. Here, the necessity to possibly exclude items from the partitioning follows from a requirement to form small tests, whenever possible.

In this paper, a globally optimal approach to (small) non-exhaustive partitioning problems will be pursued. In such an approach, the objective is to maximize the sum of some (homogeneity) function of the individual clusters/tests, without the necessity to include all objects/items in the solution. Special care in formulating this objective function is to be taken to avoid including too few or too many objects/items.

To solve this problem globally optimal, two approaches to an exact Dynamic Programming algorithm are presented. The algorithm will be applied to both a non-exhaustive clustering problem, and the construction of multiple scales from a small item bank.

## References

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