Simultaneous Parameter Estimation in Exploratory Factor Analysis by Weighted Least Squares

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- Exploratory Factor Analysis (EFA) model [Mulaik (1972)]:

$$\mathbf{Z} = \mathbf{F}\mathbf{\Lambda}' + \mathbf{U}\mathbf{\Psi} \quad ,$$

$\mathbf{F}$: $n \times k$ matrix of common factors ($k \ll p$);
$\mathbf{\Lambda}$: $p \times k$ matrix of loadings with $\text{rank}(\mathbf{\Lambda}) = k$;
$\mathbf{U}$: $n \times p$ matrix of unique factors;
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$\mathbf{F}$ and $\mathbf{U}$ are mean-centered;
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Factoring the correlation matrix

- Model correlation structure: $\mathbf{R} = \mathbf{\Lambda}\mathbf{\Lambda}' + \mathbf{\Psi}^2$.
- Factor extraction: Find a pair $\{\hat{\mathbf{\Lambda}}, \hat{\mathbf{\Psi}}\}$ which gives the best fit to the sample correlation matrix $\mathbf{C} = \mathbf{Z}'\mathbf{Z}$.
- Least squares goodness-of-fit criterion:

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- Factor scores can be obtained as a function of $\mathbf{Z}$, $\hat{\mathbf{\Lambda}}$, and $\hat{\mathbf{\Psi}}$.
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$$\hat{\mathbf{F}} = \mathbf{Z}\hat{\mathbf{\Psi}}^{-2}\hat{\mathbf{\Lambda}}\left(\hat{\mathbf{\Lambda}}'\hat{\mathbf{\Psi}}^{-2}\mathbf{C}\hat{\mathbf{\Psi}}^{-2}\hat{\mathbf{\Lambda}}\right)^{-\frac{1}{2}} \quad .$$

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Methods in the EFA literature

- Young (1940), Lawley (1942), Whittle (1952), Jöreskog (1962).
- Chapter in Horst (1965).
- McDonald (1979), [also Etezadi-Amoli and McDonald (1983)].
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Simultaneous parameter estimation by De Leeuw (2004)

- Unweighted least squares (ULS) fitting by minimizing:

$$f_{ULS} = ||\mathbf{Z} - \mathbf{F}\boldsymbol{\Lambda}' - \mathbf{U}\boldsymbol{\Psi}||_F^2$$

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- Defining block matrices $\mathbf{B} := [\mathbf{F} : \mathbf{U}]$ and $\mathbf{A} := [\mathbf{\Lambda} : \mathbf{\Psi}]$ of dimensions $n \times (p+k)$ and $p \times (p+k)$, respectively.

$$f_{ULS} = ||\mathbf{Z} - \mathbf{B}\mathbf{A}'||_F^2 = ||\mathbf{Z}||_F^2 + \text{trace}(\mathbf{A}\mathbf{A}') - 2\,\text{trace}(\mathbf{B}'\mathbf{Z}\mathbf{A}) \quad .$$

- As with the standard Procrustes problem, the minimization of f_{ULS} is equivalent to the maximization of trace$(\mathbf{B}'\mathbf{Z}\mathbf{A})$.

- For this problem an analytical solution via the Singular Value Decomposition of $\mathbf{ZA}$ exists.

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Data matrix decomposition by Weighted Least Squares

- Weighted Least Squares (WLS) loss function:

$$f_{WLS} = ||(\mathbf{Z} - \mathbf{BA}') \odot \mathbf{W}||_F^2 = \sum_{i=1}^{n} \sum_{j=1}^{p} w_{ij}^2 e_{ij}^2 \ ,$$

W: $n \times p$ matrix of non-negative weights w_{ij} for residuals e_{ij};
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Iterative Majorization by Kiers (1997), (2002)

- f_{WLS} is majorized and touched by

$$\mu(\mathbf{B},\mathbf{A}|\mathbf{B}^c,\mathbf{A}^c) = \text{constant} + w_m^2||\tilde{\mathbf{Z}} - \mathbf{BA}'||_F^2 \quad ,$$

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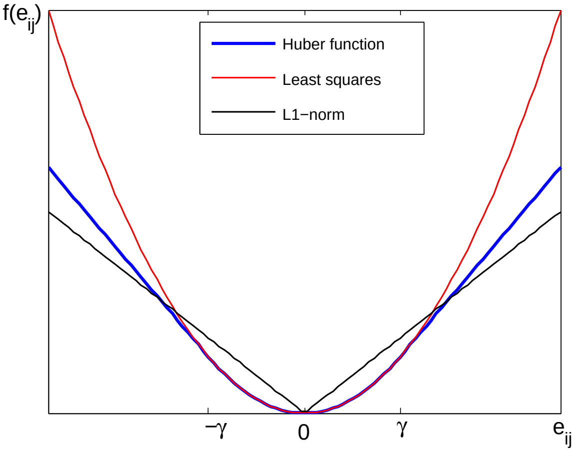
Hybrid ℓ_1-ℓ_2 goodness-of-fit criterion by Huber (1964)



- Huber's loss function:

$$f_H(\mathbf{Z} - \mathbf{BA}') = \sum_{i=1}^{n} \sum_{j=1}^{p} f_H(e_{ij}) \ ,$$

where $$f_H(e_{ij}) = \begin{cases} e_{ij}^2 & \textit{if} \quad |e_{ij}| < \gamma \ , \\ 2\gamma|e_{ij}| - \gamma^2 & \textit{if} \quad |e_{ij}| \geq \gamma \ , \end{cases}$$

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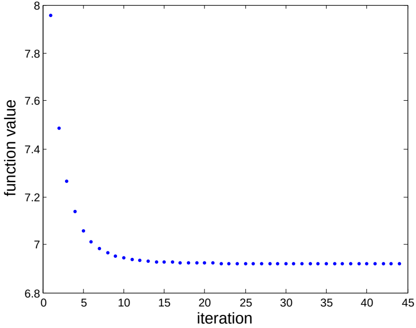
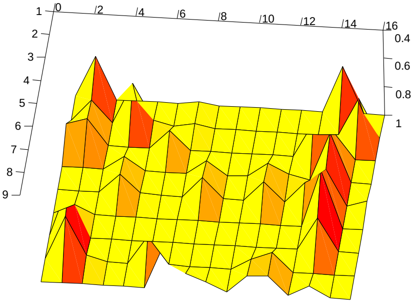
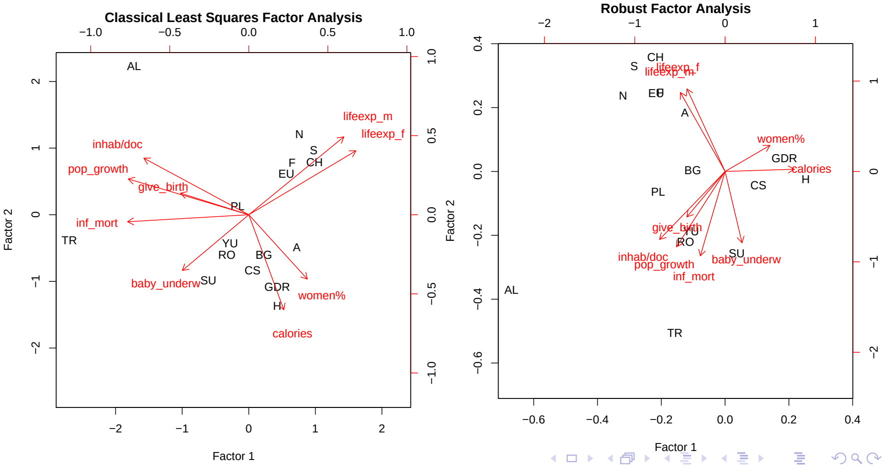
- Huber treshold [Verboon (1994)]: $\gamma = \frac{1}{3}s$, where $s = \text{median}\{|e_{ij}|\} + 4\ \text{MAD}\{|e_{ij}|\}$.

- Convergence criterion: $\epsilon = 10^{-6}$.





Biplots [Gabriel (1971), Gower and Hand (1996)]



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