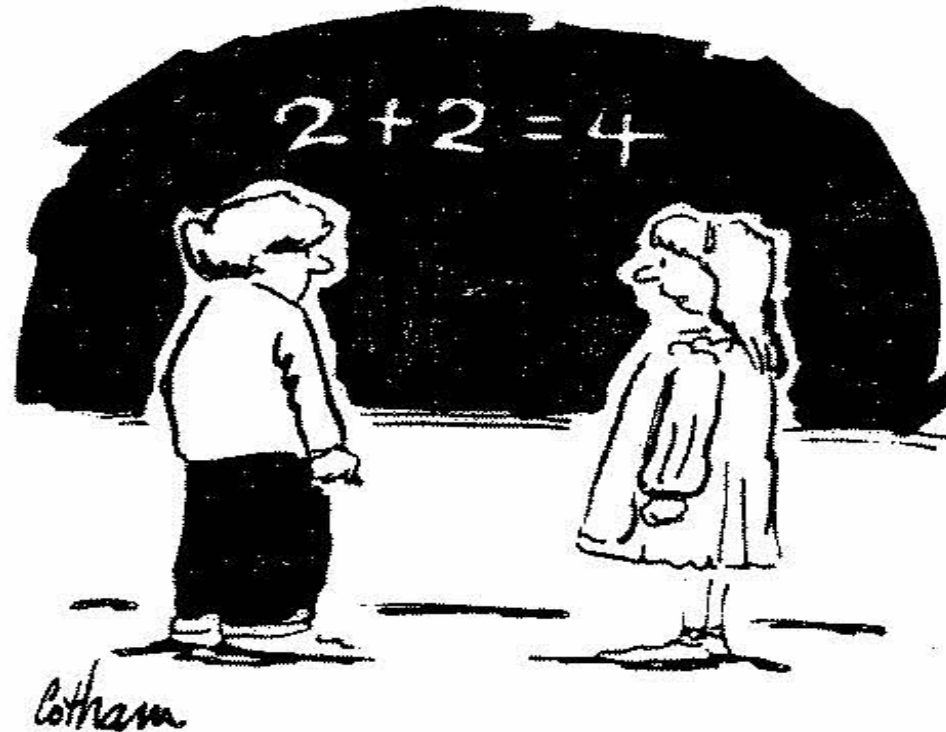


Multilevel Model Specification Tests Using the Generalized Methods of Moment (GMM) Estimation Techniques



"Everything gets much more complicated after this."

Jee-Seon Kim, Edward W. Frees, Chris M. Swoboda
University of Wisconsin at Madison

IMPS 2008

Outline

- ❑ Correlated effects between predictors and random effects
- ❑ Econometric treatment and its limitation in multilevel models
- ❑ Omitted variable tests
- ❑ Generalized methods of moments (GMM) estimator
- ❑ Simulation results (if time permits)
- ❑ An example with National Education Longitudinal Study of 1988 (NELS:88)
- ❑ Summary

Correlated Effects in Multilevel Models

- ❑ Consider a “student-teacher-school” model where schools are independent units:

$$\mathbf{y}_s = \mathbf{X}_s \boldsymbol{\beta} + \boldsymbol{\delta}_s$$

- ❑ If a predictor is correlated with $\boldsymbol{\delta}_s$, it is an endogenous variable.
- ❑ If not, it is an exogenous variable.
- ❑ Standard estimation methods for multilevel models (e.g., REML, FIML, GLS, empirical Bayes estimators) assume that all predictors are exogenous, and each of these estimators can be considered as a random effects estimator.
- ❑ The exogeneity assumption is often violated when relevant variables are omitted. Even a moderate correlation between the \mathbf{X}_s and $\boldsymbol{\delta}_s$ may result in severe bias in the estimation of $\boldsymbol{\beta}$ as well as variance components (i.e., omitted variable bias).
- ❑ In econometrics, correlated effects are examined by comparing a random effects estimator to a fixed effects estimator.

Model Specification Test in Econometrics

- bFE is robust to misspecification due to omitted person variables.
- Uses within-person deviations.
- Does not estimate person or higher-level effects.

Ho: **No misspecification**

Fixed Effects Estimator b_{FE}

Random Effects Estimator b_{RE}

- bRE assumes the most stringent form of exogeneity.
- Biased if there exist omitted person variables.
- Default and only option in most software.

Hausman (1978): Specification test for econometrics. *Econometrica*.

$$\chi^2 = \frac{(b_{FE} - b_{RE})^2}{VAR(b_{FE}) - VAR(b_{RE})}$$

If the difference is significant, reject Ho & use robust bFE.

If not, one would prefer efficient bRE.

Limited Use of Econometric Treatment in ML Models

- ❑ The Hausman test is an effective tool for panel data models, (two-level random intercept models for longitudinal data), but its use is limited for more complex models.
- ❑ A level-3 FE estimator would be biased if there exist omitted variables at level 2. The Hausman test would compare two biased estimators.
- ❑ This is not a problem of the Hausman test itself, but rather is an improper application of the test for which it is not designed.

Mean Math Scores for Subpopulations in NELS:88

Listwise Deletion (N=5278) Multiple Imputation (N=7334)

	Listwise Deletion (N=5278)	Multiple Imputation (N=7334)	
<i>Student</i>	10th grade math score	45.43	44.35
	8th grade math score	37.85	36.97
	Minority	40.43	39.11
	Caucasian	47.01	46.37
<i>Teacher</i>	Math Background	46.30	45.51
	No Math Background	43.41	42.14
	Female	44.20	42.84
	Male	44.31	43.21
	Minority	38.80	37.26
	Urban	45.36	43.33
<i>School</i>	Rural	42.53	41.47
	Public School	43.02	41.48
	Private School	52.18	51.21

Omitted Variable Tests in ML Models

- ❑ Kim and Frees (2006) provided model specification tests for general multilevel models with additional levels and random slopes.
- ❑ The method allows for FE estimators at different levels (e.g., $b_{FEteacher}$, $b_{FEschool}$) and for testing the effects of unobserved variables at different levels (e.g., $U_{teacher}$, U_{school}), jointly or separately.
- ❑ One can conduct three types of omitted variable tests.

Omitted Variable Tests	Hypothesis	Estimators compared
1. Multiple-level test	$H_0: U_{teacher}=U_{school}=0$	b_{FEt} vs. b_{RE}
2. Intermediate-level test	$H_0: U_{teacher}=0$	b_{FEt} vs. b_{FEs}
3. Highest-level test	$H_0: U_{school}=0$	b_{FEs} vs. b_{RE}

Omitted Variable Tests in a 3-Level Model for NELS:88

	<i>bF_{Et}</i>	<i>bRE</i>	
<i>Student</i>	Prior Achievement	0.85*	0.95*
	SES	0.71*	1.11*
	Female	0.14	0.17
	Minority	-0.55	-0.76*
<i>Teacher</i>	Math Background		0.53*
	Experienced		0.45
	Female		0.39*
	Minority		-0.42
<i>School</i>	Urban		-0.43
	Rural		-0.37
	School Size/100		0.02
	% Caucasian/10		0.01
	% Single Parent Homes/10		-0.10
	Public School		-0.93*

OV Test 1 H₀: U_{teacher}=U_{school}=0. *bF_{Et}* vs. *bRE*. chi-sq=143.34, df=4, p<0.01.

Omitted Variable Tests in a 3-Level Model for NELS:88

	<i>bFE_t</i>	<i>bFE_s</i>	
<i>Student</i>	Prior Achievement	0.85*	0.94*
	SES	0.71*	0.94*
	Female	0.14	0.17
	Minority	-0.55	-0.64*
<i>Teacher</i>	Math Background		0.71*
	Experienced		0.56
	Female		0.42
	Minority		-0.36
<i>School</i>	Urban		
	Rural		
	School Size/100		
	% Caucasian/10		
	% Single Parent Homes/10		
Public School			

OV Test 1 H₀: U_{teacher}=U_{school}=0. *bFE_t* vs. *bRE*. chi-sq=143.34, df=4, p<0.01.

OV Test 2 H₀: U_{teacher}=0. *bFE_t* vs. *bFE_s*. chi-sq =130.64, df=4, p<0.01.

Omitted Variable Tests in a 3-Level Model for NELS:88

	<i>bFEs</i>	<i>bRE</i>		
<i>Student</i>	Prior Achievement	0.94*	0.95*	
	SES	0.94*	1.11*	
	Female	0.17	0.17	
	Minority	-0.64*	-0.76*	
<i>Teacher</i>	Math Background	0.71*	0.53*	
	Experienced	0.56	0.45	
	Female	0.42	0.39*	
	Minority	-0.36	-0.42	
<i>School</i>	Urban		-0.43	
	Rural		-0.37	
	School Size/100		0.02	
	% Caucasian/10		0.01	
	% Single Parent Homes/10		-0.10	
			Public School	-0.93*

OV Test 1 $H_0: U_{teacher} = U_{school} = 0$. b_{FEt} vs. b_{RE} . $\chi^2 = 143.34$, $df = 4$, $p < 0.01$.

OV Test 2 $H_0: U_{teacher} = 0$. b_{FEt} vs. b_{FEs} . $\chi^2 = 130.64$, $df = 4$, $p < 0.01$.

OV Test 3 $H_0: U_{school} = 0$. b_{FEs} vs. b_{RE} .

Omitted Variable Tests in a 3-Level Model for NELS:88

		<i>Unbiased</i>	<i>Biased</i>	
		<i>bF_{Et}</i>	<i>bF_{Es}</i>	<i>bRE</i>
<i>Student</i>	Prior Achievement	0.85*	0.94*	0.95*
	SES	0.71*	0.94*	1.11*
	Female	0.14	0.17	0.17
	Minority	-0.55	-0.64*	-0.76*
<i>Teacher</i>	Math Background		0.71*	0.53*
	Experienced		0.56	0.45
	Female		0.42	0.39*
	Minority		-0.36	-0.42
<i>School</i>	Urban			-0.43
	Rural			-0.37
	School Size/100			0.02
	% Caucasian/10			0.01
	% Single Parent Homes/10			-0.10
	Public School			-0.93*

OV Test 1 $H_0: U_{teacher} = U_{school} = 0$. $b_{F_{Et}}$ vs. b_{RE} . $\chi^2 = 143.34$, $df = 4$, $p < 0.01$.

OV Test 2 $H_0: U_{teacher} = 0$. $b_{F_{Et}}$ vs. $b_{F_{Es}}$. $\chi^2 = 130.64$, $df = 4$, $p < 0.01$.

OV Test 3 $H_0: U_{school} = 0$. $b_{F_{Es}}$ vs. b_{RE} is not meaningful as $b_{F_{Es}}$ is biased.

Although $b_{F_{Et}}$ is unbiased, it does not provide estimates at levels 2 and 3.

Generalized Methods of Moment (GMM) Estimator

- Kim and Frees (2007) presented a GMM estimator for multilevel models.

$$\mathbf{b}_{\text{GMM}} = (\mathbf{X}' \mathbf{W} P(\mathbf{H}) \mathbf{W} \mathbf{X})^{-1} \mathbf{X}' \mathbf{W} P(\mathbf{H}) \mathbf{W} \mathbf{y}$$

\mathbf{H} is a set of instruments (def: exogenous variables used for estimating β)
 $P(\mathbf{H}) = \mathbf{H}(\mathbf{H}'\mathbf{H})^{-1}\mathbf{H}'$ is the projection spanned by \mathbf{H} . \mathbf{W} : weight matrix.

- In instrumental variable (IV) methods, one uses additional variables outside of the model (i.e., instruments) when some \mathbf{X} are endogenous.
- In contrast, the multilevel GMM approach can compose instruments \mathbf{H} using functions of \mathbf{X} without external variables.
- Thus, in this “internal” IV approach, external variables can be included in \mathbf{H} but are not required.
- Also, **bgmm** encompasses the fixed effects (FE) and random effects (RE) estimators as special cases and provides additional options.

FE, RE, and the GMM Estimator as a Middle Ground

- ❑ The FE estimator is obtained:
 - based on the idea that all predictors might be endogenous
 - by using only within-group deviations as instruments
- ❑ The RE estimator is obtained:
 - based on the assumption that all predictors are exogenous
 - by using both within-group deviations and group means as instruments
- ❑ The GMM estimator takes a middle-of-the-road approach:
 - regards some predictors as endogenous and the rest as exogenous
 - uses both within and between group information of exogenous variables
 - use only within-group information of endogenous variables as instruments
 - The GMM estimator can be as robust as the FE estimator and provides estimates at all levels.

Estimator Continuum (Robust to Efficient)

most robust

most efficient

	<i>bFE_t</i>	<i>bGMM_t</i>	<i>bFE_s</i>	<i>bGMM_s</i>	<i>bRE</i>
<i>Student</i>	Prior Achievement	0.85*		0.94*	0.95*
	SES	0.71*		0.94*	1.11*
	Female	0.14		0.17	0.17
	Minority	-0.55		-0.64*	-0.76*
<i>Teacher</i>	Math Background			0.71*	0.53*
	Experienced			0.56	0.45
	Female			0.42	0.39*
	Minority			-0.36	-0.42
<i>School</i>	Urban				-0.43
	Rural				-0.37
	School Size/100				0.02
	% Caucasian/10				0.01
	% Single Parent Homes/10				-0.10
	Public School				-0.93*

Recall that only **bFE_t** is unbiased and both **bFE_s** and **bRE** are biased. Although unbiased, **bFE_t** does not provide estimates at higher levels. We can obtain GMM estimators and create an estimator continuum.

Estimator Continuum (Robust to Efficient)

most robust

most efficient

	<i>bFE_t</i>	<i>bGMM_t</i>	<i>bFE_s</i>	<i>bGMM_s</i>	<i>bRE</i>	
<i>Student</i>	Prior Achievement	0.85*	0.85*	0.94*	0.94*	0.95*
	SES	0.71*	0.71*	0.94*	0.94*	1.11*
	Female	0.14	0.12	0.17	0.17	0.17
	Minority	-0.55	-0.55	-0.64*	-0.68*	-0.76*
<i>Teacher</i>	Math Background		0.58*	0.71*	0.52*	0.53*
	Experienced		0.59	0.56	0.47	0.45
	Female		0.39	0.42	0.40*	0.39*
	Minority		0.16	-0.36	-0.27	-0.42
<i>School</i>	Urban		0.18		-0.28	-0.43
	Rural		-0.54		-0.39	-0.37
	School Size/100		0.08*		0.04	0.02
	% Caucasian/10		0.52*		0.12	0.01
	% Single Parent Homes/10		-0.05		-0.08	-0.10
Public School		-1.94*		-1.07*	-0.93*	

Prior achievement, SES, and % Caucasian might be endogenous.
 We can obtain two GMM estimators at the teacher and school levels. Within teacher (school) deviations and teacher (school) means as instruments.

Estimator Continuum (Robust to Efficient)

		<i>Unbiased</i>		<i>Biased</i>		
		<i>bFET</i>	<i>bGMMt</i>	<i>bFEs</i>	<i>bGMMs</i>	<i>bRE</i>
<i>Student</i>	Prior Achievement	0.85*	0.85*	0.94*	0.94*	0.95*
	SES	0.71*	0.71*	0.94*	0.94*	1.11*
	Female	0.14	0.12	0.17	0.17	0.17
	Minority	-0.55	-0.55	-0.64*	-0.68*	-0.76*
<i>Teacher</i>	Math Background		0.58*	0.71*	0.52*	0.53*
	Experienced		0.59	0.56	0.47	0.45
	Female		0.39	0.42	0.40*	0.39*
	Minority		0.16	-0.36	-0.27	-0.42
<i>School</i>	Urban		0.18		-0.28	-0.43
	Rural		-0.54		-0.39	-0.37
	School Size/100		0.08*		0.04	0.02
	% Caucasian/10		0.52*		0.12	0.01
	% Single Parent Homes/10		-0.05		-0.08	-0.10
	Public School		-1.94*		-1.07*	-0.93*

We found two out of five estimators are unbiased for the three-level model for NELS:88. Therefore, we chose **bGMMt**, which is the most efficient among unbiased estimators.

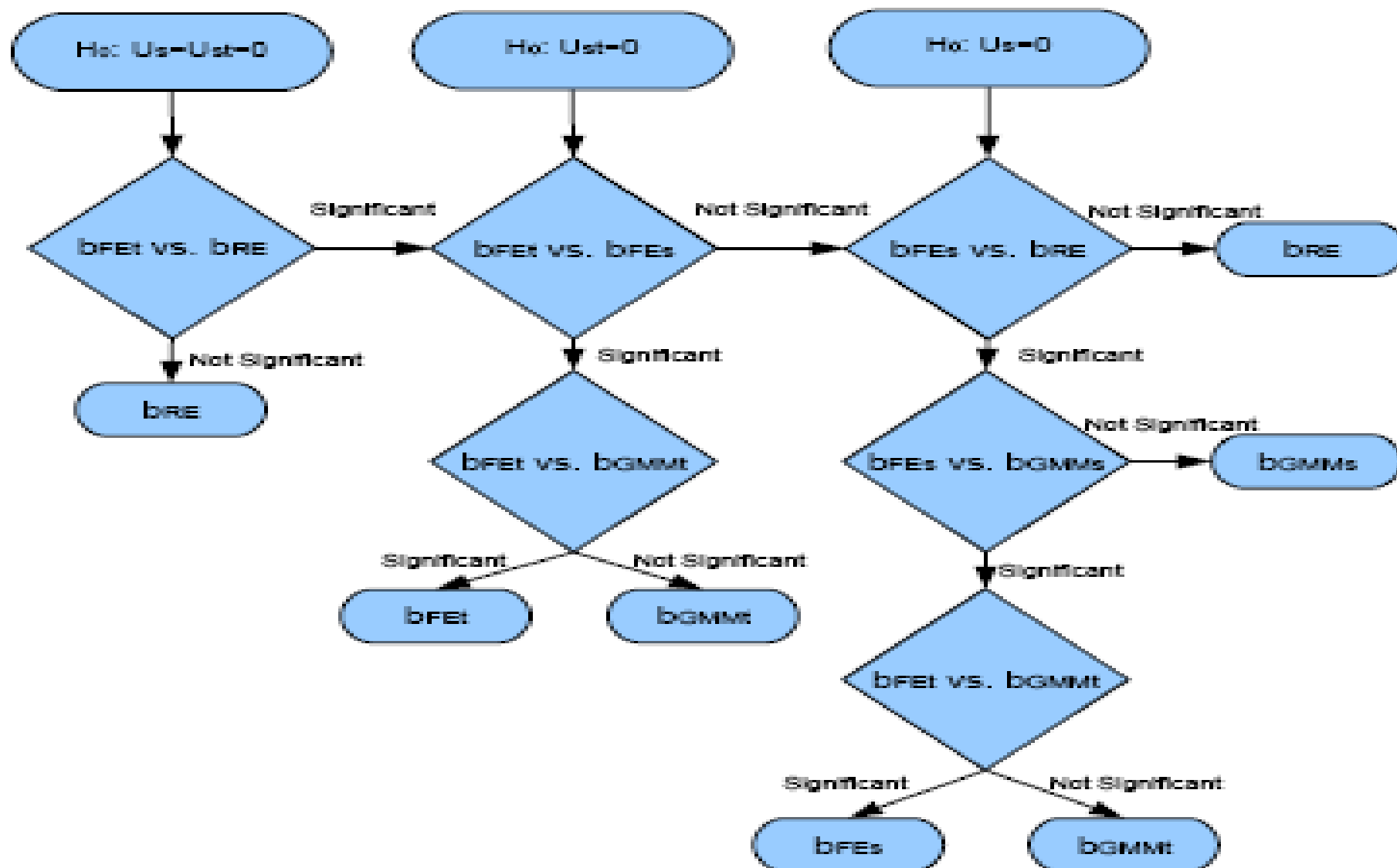
Guideline for Omitted Variable Tests and Optimal Estimator

Kim & Swoboda (2008)

1. Multiple-Level Test

2. Intermediate-Level Test

3. Highest-Level Test



Individual Coefficient Tests

	<i>bGMMt</i>	<i>bRE</i>	chi-sq (<i>df</i> =1)
<i>Student</i>	Prior Achievement	0.85*	108.70**
	SES	0.71*	9.85**
	Female	0.12	0.42
	Minority	-0.55	1.11
<i>Teacher</i>	Math Background	0.58*	0.20
	Experienced	0.59	0.73
	Female	0.39	0.00
	Minority	0.16	3.55
<i>School</i>	Urban	0.18	5.04*
	Rural	-0.54	1.15
	School Size/100	0.08*	11.69**
	% Caucasian/10	0.52*	14.20**
	% Single Parent Homes/10	-0.05	0.44
	Public School	-1.94*	23.72**

Summary

- ❑ This study presents statistical methods for handling endogeneity in multilevel models due to omitted variables.
- ❑ In the social sciences, researchers cannot always design a study that is appropriate for addressing misspecification issues (e.g., observational or quasi-experimental studies).
- ❑ Our NELS:88 analysis reveals that under the exogeneity assumption, the random effects estimator underestimates the effects of school variables (e.g., school size, % Caucasian), while overestimating the effects of student variables (e.g., minority, SES, prior achievement).
- ❑ Thus, the unbiased GMM estimator is recommended.
- ❑ Multilevel data contain rich information, and the GMM estimation technique exploits the hierarchical structure for testing correlated effects and for obtaining alternative robust estimators when some predictors are endogenous.

References

Kim & Frees (2006). Omitted variables in multilevel models.
Psychometrika

Kim & Frees (2007). Multilevel modeling with correlated effects.
Psychometrika

Kim & Swoboda (in press). Handling omitted variable bias.
In J.J. Hox & J.K. Roberts (Eds.), *Handbook of advanced multilevel analysis*.

Email to Jee-Seon Kim at jeeseonkim@wisc.edu for reprint.